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Chiral expansion for low-energy η-nucleon interactions with explicit resonance

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We consider the process of low-energy η-nucleon interactions in the framework of heavy baryon chiral perturbation theory with the $S_{11}(1535)$ resonance treated as an independent field. We include leading and next-to-leading terms in chiral expansion of the amplitudes of the pion induced η-meson production and elastic η-nucleon scattering.

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The ideas of effective field theory applied to the system of interacting baryons and mesons by means of heavy baryon chiral perturbation theory (HBCPT) [1] have been proven to be successful in describing a variety of low-energy processes. HBCPT involves the expansion in the parameter Q/Λ where Q is a typical momentum involved and Λ is a ''heavy mass scale.'' The chiral expansion is supplemented by the counting rules [2] allowing one to estimate the contribution of a certain graph or several graphs in a given chiral order. However, this method cannot straightforwardly be generalized to the case where the corresponding matrix elements exhibit a pole close to threshold so that the chiral expansion is divergent. This singularity can either be a virtual bound state, as in the case of the ${}^1S_0$ nucleon-nucleon scattering, or a resonance when the low-energy K^-N scattering is considered. One of the possible ways to treat self-consistently the ${}^1S_0$ nucleon-nucleon scattering is to introduce the effective ''dibaryon'' field [3] in the chiral Lagrangian which, in some sense, imitates the presence of the virtual bound state (sometimes called ''virtual deuteron''). A similar idea was explored in Refs. [4,5] to study the process of the low-energy K^-N interaction where the $\Lambda(1405)$ resonance was introduced in the effective chiral Lagrangian as an independent field. In the present paper we use this approach to consider the low-energy ηN interaction. The main dynamical feature of such a system is an existence of the $S_{11}(1535)$ resonance near threshold so the method developed to treat the K^-N scattering could be applicable in this case too. One notes that there is another method to treat such a system [6]. It is based on the Weinberg type approach [7] where the tree level diagrams are interpreted as some effective potential which is to be iterated to all orders. The $S_{11}(1535)$ resonance can exhibit itself as the pole of the T matrix obtained. However, in such an approach the chiral counting rules are less transparent since only ladder diagrams are summed up when the Lippmann-Schwinger equation is solved. Moreover, the results depend to some extent on the type of regulator used. The method of treating the resonance as an independent field is, in a sense, closer in spirit to the standard HBCPT. However, the problem with this latter approach is that the effective low-energy constants at the $0(p^3)$ order are poorly known so the fully consistent calculations explicitly including loops are not feasible at present. We discuss this point in more detail below. The effective chiral Lagrangian we use can schematically be written as

$$\mathcal{L}=\mathcal{L}^{(1)}+\mathcal{L}^{(2)}. \tag{1}$$

Here $\mathcal{L}^{(1)}$ is the leading order term in the chiral expansion which is given by

$$\mathcal{L}^{(1)}=\frac{i}{8f^2}\mathrm{Tr}(\bar{B}[[\phi,\partial_0\phi],B])+\mathrm{Tr}(B^*(v\cdot A)B)+\mathrm{H.c.}, \tag{2}$$

and the next-to-leading order term of the effective Lagrangian is

$$\begin{aligned}\mathcal{L}^{(2)}=&\frac{i}{2M_0}\mathrm{Tr}(\bar{B}[(v\cdot D)^2-D^2]B)+b_D\mathrm{Tr}(\bar{B}\{\chi_+,B\})+b_F\mathrm{Tr}(\bar{B}[\chi_+,B])+b_0\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(\chi_+)+d_D\mathrm{Tr}(\bar{B}\{A^2+(v\cdot A)^2,B\})\\&+d_F\mathrm{Tr}(\bar{B}[A^2+(v\cdot A)^2,B])+d_0\mathrm{Tr}(\bar{B}B)\mathrm{Tr}(A^2+(v\cdot A)^2)+d_1[\mathrm{Tr}(\bar{B}A_\nu)\mathrm{Tr}(A^\nu B)+\mathrm{Tr}(\bar{B}(v\cdot A))\mathrm{Tr}((v\cdot A)B)]\\&+d_2\mathrm{Tr}[\bar{B}(A_\nu BA^\nu+(v\cdot A)B(v\cdot A))].\end{aligned} \tag{3}$$

As usual we defined the covariant derivative as $D^\nu B=\partial^\nu B+[\Gamma^\nu,B]$ with a chiral connection Γ^ν and axial operator $A_\nu=-(1/2f)\partial_\nu\phi$. The matrix χ_+ is given by

$$\chi_+=-\frac{1}{4f^2}\{\phi,\{\phi,\chi\}\}. \tag{4}$$

Here B and ϕ are baryon and meson field operators, respectively. The normalizations used are the same as those assumed in Ref. [6]. We denoted χ the diagonal meson mass matrix with nonzero diagonal elements $(m_\pi^2,m_\pi^2,2m_K^2$

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$-m_\pi^2$). The operator describing the S_{11} resonance field is contained in the second term of the expression for $\mathcal{L}^{(1)}$. We defined B^* the corresponding field operator. The low-energy effective constants should be determined from experiment. The numerical values of the constants $\{b\}$ were extracted following the procedure used in [6]. The parameters b_d and b_F are related to the baryon mass splitting. Using $SU(3)$ expressions relating the baryon masses one can get $b_D = 0.06\ \mathrm{GeV}^{-1}$ and $b_F = -0.22\ \mathrm{GeV}^{-1}$. The value of b_0 is obtained from the πN sigma term which is related to b_0 as follows:

$$\sigma_{\pi N} = -2m_\pi^2(b_D + b_F + 2b_0). \tag{5}$$

Using this relation we get $b_0 = -0.34\ \mathrm{GeV}^{-1}$.

The constants $\{d's\}$ related to the double derivative terms in the chiral Lagrangian were determined in Ref. [5] from the analysis of K^-p scattering at low energies. Let us briefly describe the procedure adopted in [5]. There are six channels $(\pi\Sigma, \pi\Lambda, KN)$ altogether which can participate in the process of K^-p interaction at energies close to threshold. There are also two expressions, relating isospin-even πN S-wave scattering length $a^+_{\pi N}$ and isospin zero kaon-nucleon S-wave scattering length a^0_{KN} with some of the low-energy constants

$$4\pi\left(1+\frac{m_\pi}{M_N}\right)a^+_{\pi N} = \frac{m_\pi^2}{f^2}\left(d_D + d_F + 2d_0 - 2b_D - 2b_F - 4b_0 - \frac{g_A^2}{4M_N}\right) \tag{6}$$

and

$$4\pi\left(1+\frac{m_K}{M_N}\right)a^0_{KN} = \frac{m_K^2}{f^2}\left(-2d_F + 2d_0 - d_1 + 4b_F - 4b_0 - \frac{D}{M_N}(F - D/3)\right), \tag{7}$$

where F and D are the standard SU(3) coupling constants satisfying the relation $D+F=1.26$. Using these relations two effective coupling constants can be determined. The remaining constants were fixed using the experimental data on K^-p reaction with the different final states. There are six possible final states altogether so that after all the effective constants were fixed three independent theoretical predictions were obtained.

The other parameters involved in the present calculations are the mass and width of the S_{11} resonance. We used the values 1535 MeV and 185 MeV, respectively. Unlike the mass of the S_{11} resonance, which is determined to a reasonable accuracy, the width of the resonance is still quite a poorly known parameter. Different analyses give the values of the total width ranging from 100 MeV up to 250 MeV. The situation is even worse with the partial widths related to the decay channels πN, ηN, and $\pi\pi N$. We adopted the value of the total width lying somewhere in between the extreme points. One notes that formally we calculate the amplitude of the ηN interaction to the next-to-leading order. In this order the amplitude is still a real quantity. Taking into account the resonance width corresponds to the inclusion of the loop corrections, which are of order $O(p^3)$, so our approach is not fully consistent. However, it does not seem feasible at present to carry out the complete one-loop calculations with the $SU(3)$ chiral Lagrangians since the numerical values of most of the effective constants are practically not known. Therefore, we assume that the most important part of the one-loop corrections in the vicinity of the resonance is due to the formation and subsequent decay of the S_{11} resonance so that these corrections can effectively be accounted for by including explicitly in the calculations the width of the resonance. One notes, in addition, that the imaginary part of the amplitude, which is assumed to be determined by the resonance width, is needed to satisfy unitarity, at least approximately. We used the values $g_\pi = 0.55$ and $g_\eta = 1.7$ for the corresponding coupling constants. The expression for the amplitude of the pion-induced η production up to next-to-leading order but with the nonzero width of the S_{11} resonance is

$$T_{\pi\eta} = F^{-2}\left[\frac{1}{\sqrt{6}}E_\eta E_\pi(d_D + d_F + 2d_2) - 4(b_d + b_F)m_\pi^2 + \frac{2}{\sqrt{6}}g_\pi g_\eta \frac{E_\eta E_\pi}{E_{\mathrm{tot}} - M^* - i\Gamma/2}\right]. \tag{8}$$

In Fig. 1 the results of our theoretical calculations are compared with the experimental data [8]. The agreement with the experimental data is quite reasonable even in the energy region lying beyond the vicinity of the resonance and comparable to that obtained in more phenomenological approaches [9]. At higher energies the nonresonant terms become more important. Thus, it seems possible that if we parametrize in some way the contribution of the next $S_{11}(1620)$ resonance and calculate the nonresonant background using HBCPT, a reasonable description of the $\pi N \to \eta N$ reaction in the relatively large energy region can be obtained. However, we did not try this possibility in the present paper. One notes that the quite large experimental errors make it difficult to estimate the importance of the neglected nonresonant one-loop corrections whose contributions are expected to be of order of the experimental errors or smaller.

Let us now turn to the issue of the ηN scattering length. Its value is usually obtained from the process of the photo-induced η production [10]. There is also a potential possibility to extract this quantity from the η-nucleus bound states [11]. However, the numerical value of the η-nucleon scattering length is not very well known. Its real and imaginary parts vary from 0.27 fm up to 0.98 fm and from 0.19 fm up to 0.4 fm, respectively [9–11]. The expression for the ηN scattering amplitude, obtained from chiral effective Lagrangian is

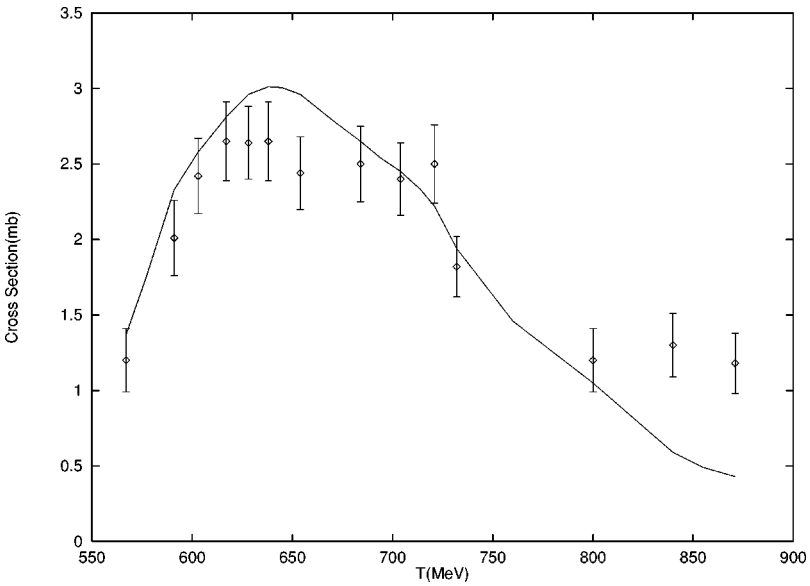

FIG. 1. Results of the theoretical calculations of the total cross section compared with experimental data [8] for the pion induced η-meson production.

$$T_{\eta\eta}=F^{-2}\left[\left(b_d-b_F+\frac{2}{3}b_0\right)m_\pi^2+\left(\frac{8}{3}(b_F-b_d)-6b_0\right)m_k^2 + \left(\frac{1}{12}(5d_D-3d_F)+d_0-\frac{1}{3}d_2\right)E_\eta^2 + \frac{1}{\sqrt{12}}g_\eta^2\frac{E_\eta^2}{E_{\rm tot}-M^*-i\Gamma/2}\right]. \quad (9)$$

For the numerical value of the ηN scattering length it gives

$$a_{\eta N}=(0.54+i0.49). \quad (10)$$

The real part of the ηN scattering length agrees well with the ''averaged'' value, obtained from phenomenological analysis. The imaginary part is somewhat larger than the typical ''averaged'' one. This is mainly due to the neglection of the nonresonant loop corrections, which constitute some piece of the imaginary part of the ηN scattering length. This is probably the case in the pion-induced η production reaction too. However, as we already mentioned, the experimental errors are larger than the expected nonresonant one-loop corrections.

There are several points where the present approach can be improved. First, the full treatment of the one-loop corrections should be done. Secondly, the partial decay ratios should also be determined from the same Lagrangian. Thirdly, the same Lagrangian can be used to calculate photo- and electro-production of the η meson, where the new experimental data have recently become available. It may help to fix better the low-energy effective constants. The results of the η-meson photoproduction calculations will be reported in future publications.

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